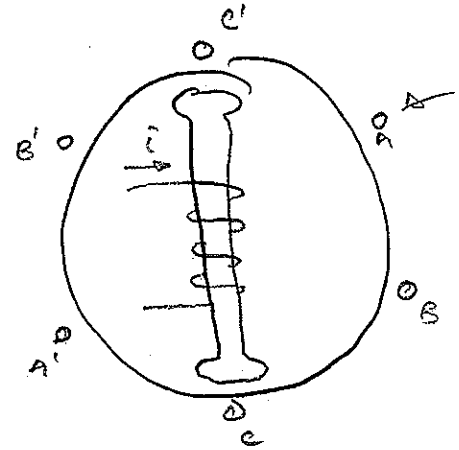
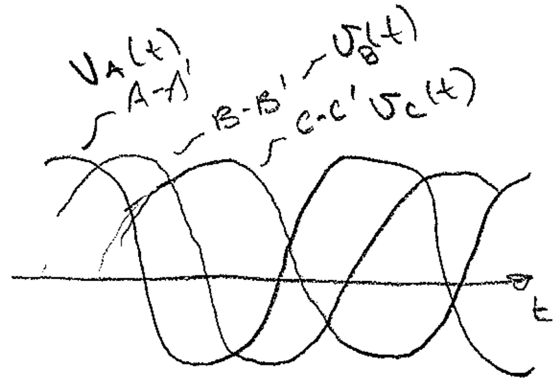
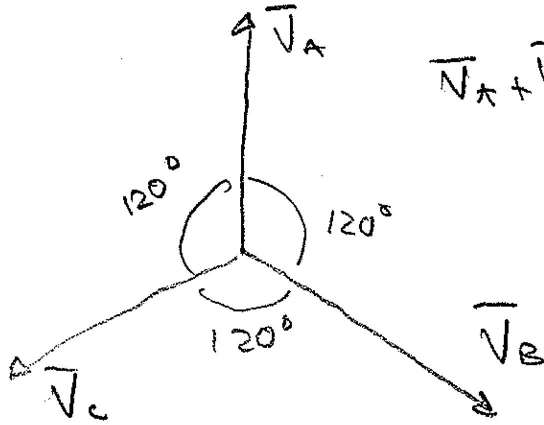
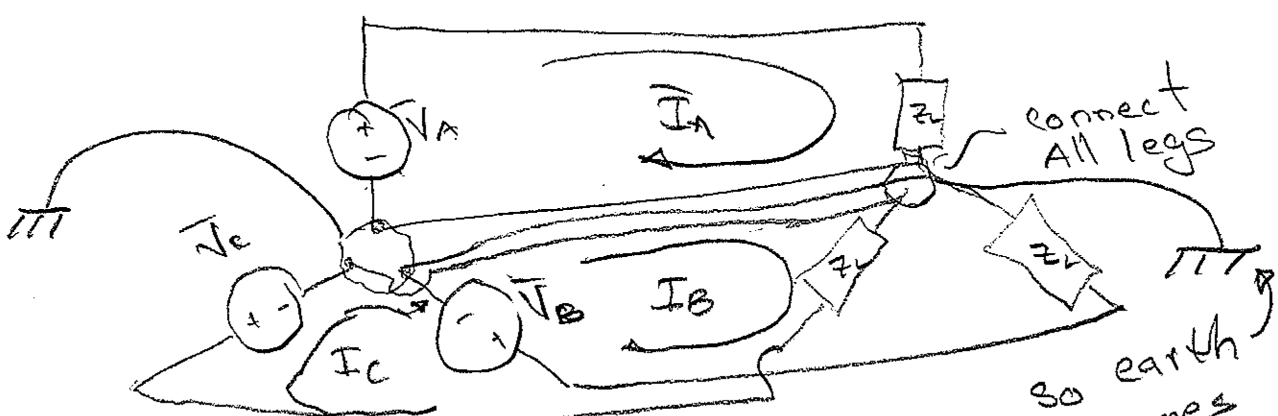
Three Phase







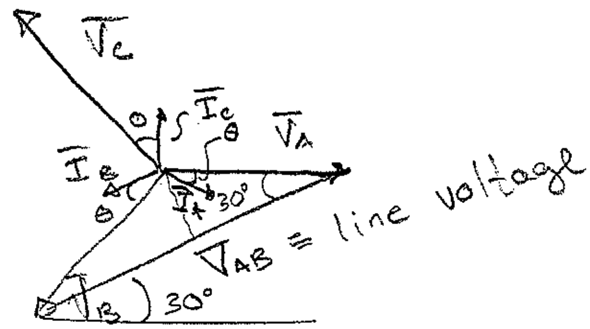
$$\bar{V}_A + \bar{V}_B + \bar{V}_C = 0$$

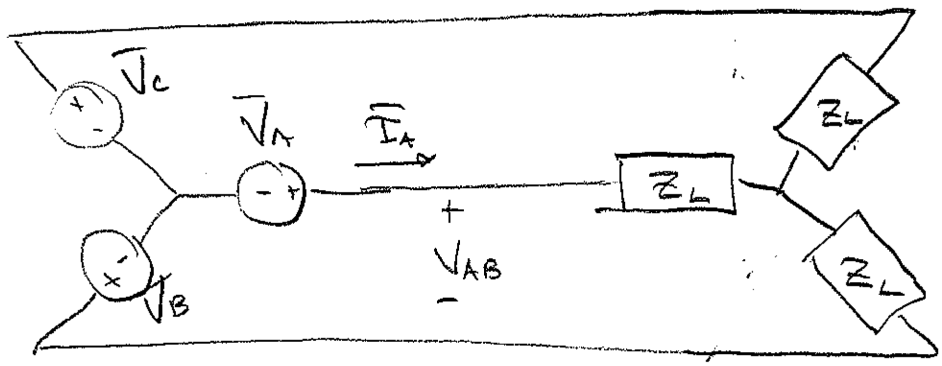


so earth becomes the return leg.

$$\bar{I}_A = \frac{\bar{V}_A}{Z_L} \; ; \; \bar{I}_B = \frac{\bar{V}_B}{Z_L} \; ; \; \bar{I}_C = \frac{\bar{V}_C}{Z_L}$$

Note: $\bar{I}_A + \bar{I}_B + \bar{I}_C = 0$ if Z_L's are all the same so the return leg isn't actually needed if it is balanced. Balanced Y-Y connection.





$$\left|\overline{V}_{AB}\right| = 2\cos 30° \left|\overline{V}_A\right| = \sqrt{3}\left|\overline{V}_A\right|$$

$$\overline{V}_{AB} = \overline{V}_A e^{j30°}$$

For Y-Y the line currents equal the phase currents.

$$P_A = \left|\overline{V}_A\right|\left|\overline{I}_A\right| \cos\theta$$

$$P = P_A(3) = 3\overline{V}_A \overline{I}_A \cos\theta = 3\frac{\left|\overline{V}_{AB}\right|}{\sqrt{3}} \overline{I}_A \cos\theta$$

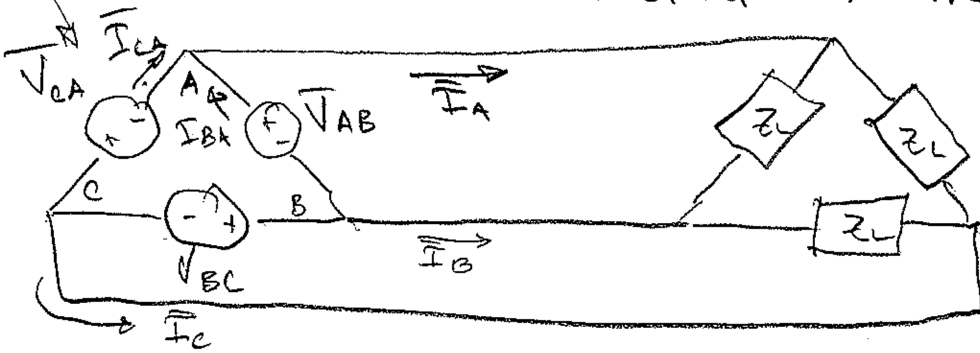
$$= \sqrt{3}\left|\overline{V}_{AB}\right|\left|\overline{I}_{AB}\right| \cos\theta$$

$$Q = \sqrt{3}\left|\overline{V}_{AB}\right|\left|\overline{I}_{AB}\right| \sin\theta$$

Note: This didn't go boom!

$$S = P + jQ = \sqrt{3}\left|\overline{V}_{AB}\right|\left|\overline{I}_A\right| e^{j\theta}$$

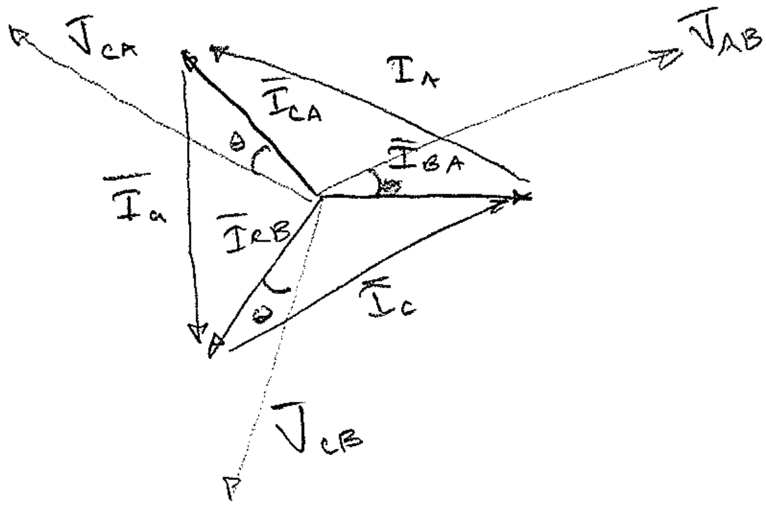
The Δ-Δ is a dual of the Y-Y.



$\overline{V}_{CA} + \overline{V}_{BC} + \overline{V}_{AB} = 0$

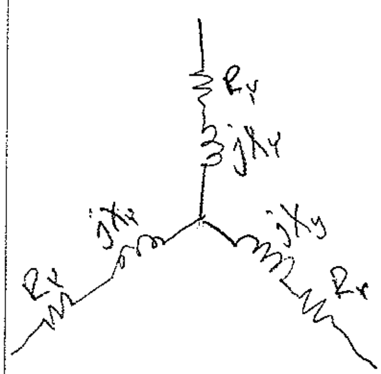
Now the line to line voltages are the same as the phase voltages. The line currents are given by:

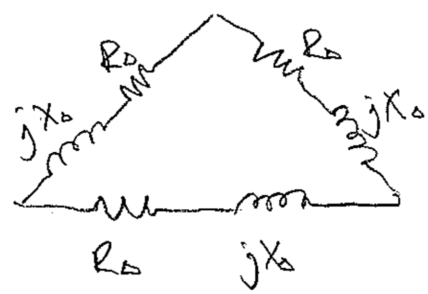
$$\bar{I}_A = \bar{I}_{CA} + \bar{I}_{BA} = \sqrt{3}\,\bar{I}_{CA}\,e^{j30°}$$



$$\underbrace{S = \sqrt{3}\,\bar{V}_\ell \bar{I}_\ell e^{j\theta}}_{}$$

for either Y-Y or Δ-Δ.





If the power is the same, $P = \sqrt{3}\,V_\ell I_\ell \cos\theta$, and they are connected to the same generator, then V_ℓ, I_ℓ are the same. Assume θ is the same.

$$Z_Y = \frac{V_{\phi Y}}{I_{\phi Y}} = \frac{V_\ell}{\sqrt{3}\,I_\ell} \qquad Z_\Delta = \frac{V_\ell}{I_{\phi\Delta}} = \frac{V_\ell\sqrt{3}}{I_\ell}$$

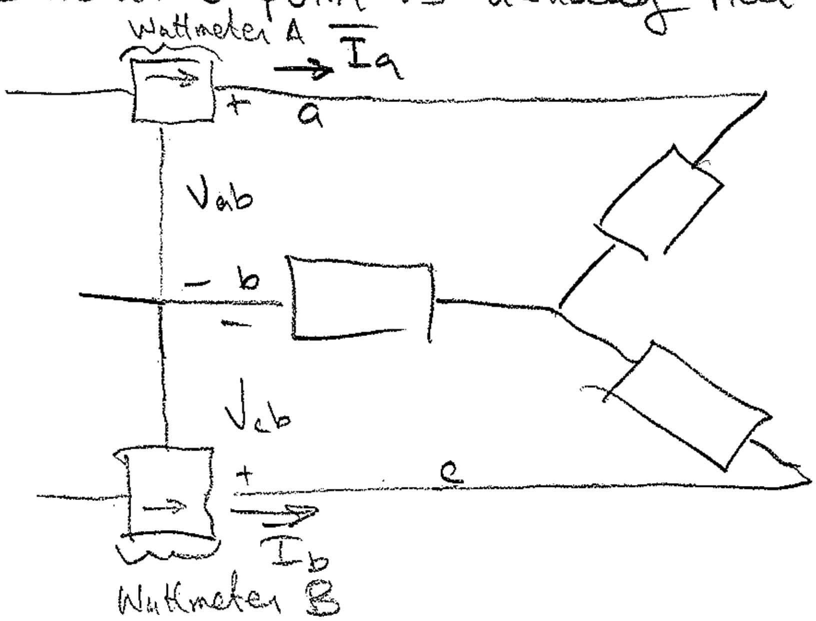
$$\frac{Z_Y}{Z_\Delta} = \frac{1}{3}$$

If we want Z_{EQ} to be the same, then θ is the same.

Rod did P 1-27 as an example

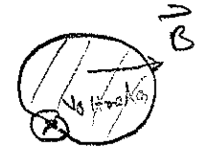
How to Measure Power in 3-Phase using two Wattmeters

The neutral point is usually not available



Think of b as the "ground" leg.

$P_{total} = P_A + P_B$



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{v} \times \vec{B})$$

$$\int \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{s} + \oint (\vec{v} \times B) \cdot d\vec{l}$$

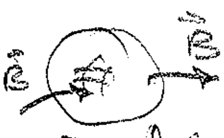
Faraday's Law

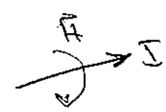
$$\nabla \cdot \vec{D} = \rho$$

$$\oint \vec{D} \cdot d\vec{S} = \int \rho \, dv$$

Gauss

$\nabla \cdot \vec{B} = 0$ or $\oint \vec{B} \cdot d\vec{s}$

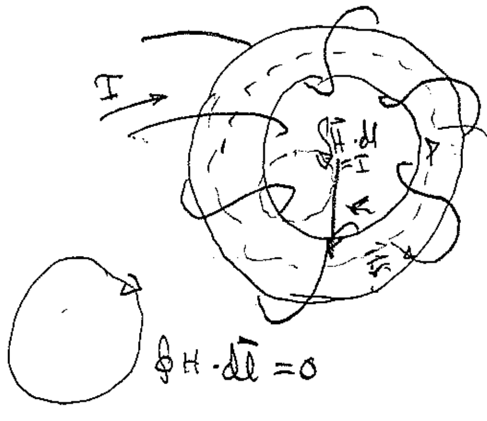




$$\nabla \times \vec{H} = \mu \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{or} \quad \oint \vec{H} \cdot d\vec{l} = \int \left(J + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S}$$

Ampere's Law

Review of Electromagnetics



N turns

$$\oint \vec{H} \cdot d\vec{l} = NI$$

Assume $\vec{H}$ is constant along the path.

$$H(2\pi r) = NI \quad \Rightarrow \quad \vec{H} = \frac{NI}{2\pi R}\hat{a}_\phi = \frac{NI}{l}\hat{a}_\phi$$

H $\left(\frac{\text{Ampere turns}}{\text{meter}}\right)$ $\quad \vec{B} = \mu \vec{H}$ $\left(\frac{\text{Webers}}{\text{square meter}}\right) = (\text{tesla})$

or (Gauss) or (lines)

Now find the flux going through the core. Assume $|\vec{B}|$ is constant over the cross section.

$$\Phi = \int \vec{B} \cdot d\vec{s} = B \underbrace{\pi r^2}_{A} = \frac{\mu N I}{l} A = \frac{NI}{\left(\frac{l}{\mu A}\right)} = \Phi$$

$$\mathcal{R} = \frac{l}{\mu A} \quad \text{like} \quad R = \frac{l}{\sigma A}$$

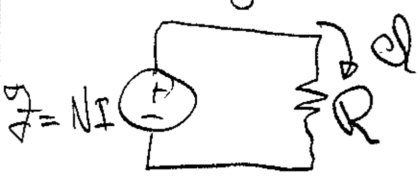
eluctance

Flux $\Phi \rightarrow$ Current

$NI \rightarrow$ Voltage

Magnetomotive Force

$$\underbrace{\Phi = \frac{mmf}{\mathcal{R}}}_{\text{Like Ohm's Law}} = \frac{\mathcal{F}}{\mathcal{R}}$$



$\nabla \cdot \vec{B} = 0 \Rightarrow$ KCL equivalent

$$v = L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$Li = N\phi \qquad \Longrightarrow L = \frac{N\phi}{i}$$

Rod has the styrofoam toroid setup.

$i = 1A$

$N = 410$

$R = 10.5$ cm

$r = 2$ cm

$\mathcal{F} = NI = 410$

$H = \frac{NI}{2\pi R} = 621.5 \ \frac{\text{Ampturns}}{\text{meter}}$

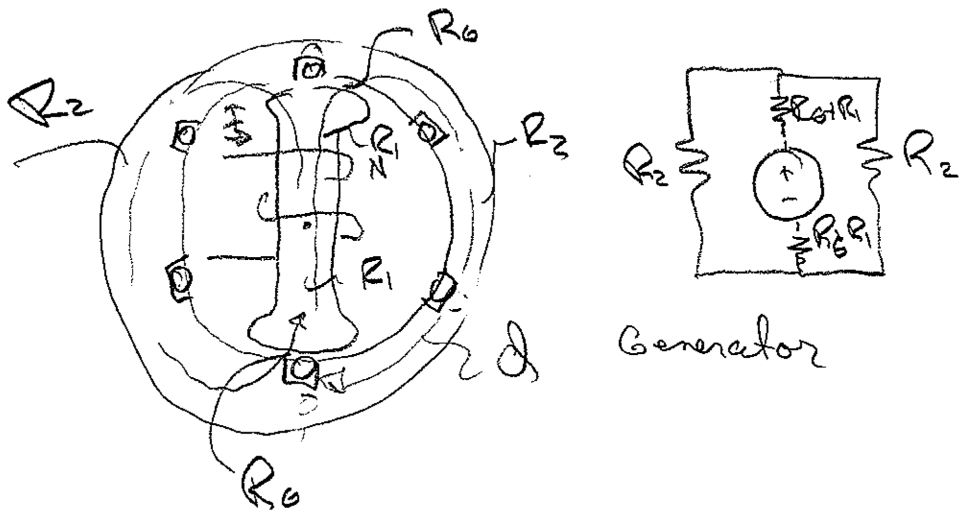
$B = \mu_0 H = 4\pi\times10^{-7}(621.5) = 7.8\times10^{-4}$ T

$= 7.81$ Gauss

Using the Gauss meter we measure, 7.5 Gauss.

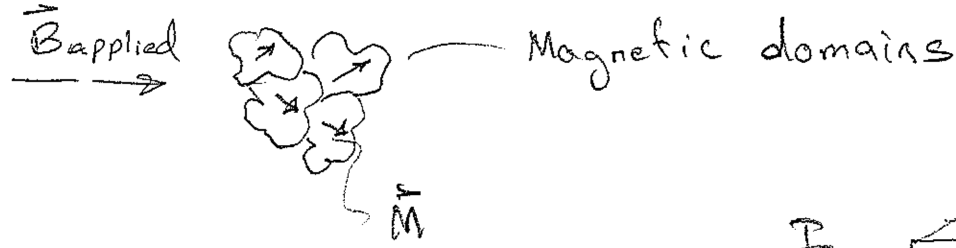
$$L = \frac{N\phi}{I} = \frac{410\,(7.8\times10^{-4})\,\pi\,(.02)^2}{1} = .402 \text{ mH}$$

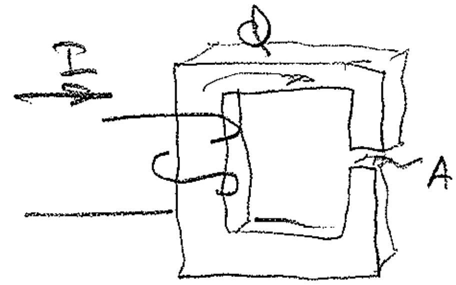
Wavetek measures $L = .38$ mH.

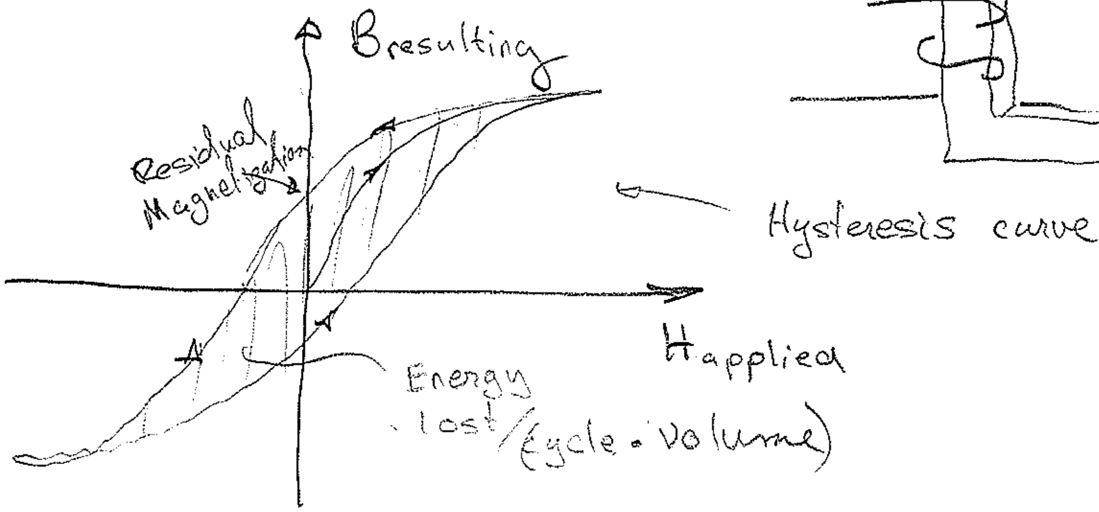


Generator

Rod showed how to magnetize a paperclip







$\oint \vec{H} \cdot d\vec{l} = NI \quad \Rightarrow H = \frac{NI}{l} \qquad \Phi = \int \vec{B} \cdot d\vec{s} = BA$

Applied

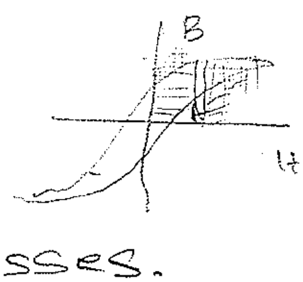
$\vec{B} \neq \mu \vec{H}$ because it is not linear

$\frac{1}{2} \int \vec{B} \cdot \vec{H} \, dv \equiv E$

$\frac{1}{2} L i^2 = \frac{1}{2} \left(\frac{N\Phi}{i} \right) i^2 = \frac{1}{2} N i \Phi = \frac{1}{2} H l B A$

$= \frac{1}{2} B H \text{ (volume)}$

$d\left(\frac{1}{2} BH\right) = \frac{1}{2} B \, dH + \frac{1}{2} H \, dB$



This covers core losses.

Eddy Current Losses

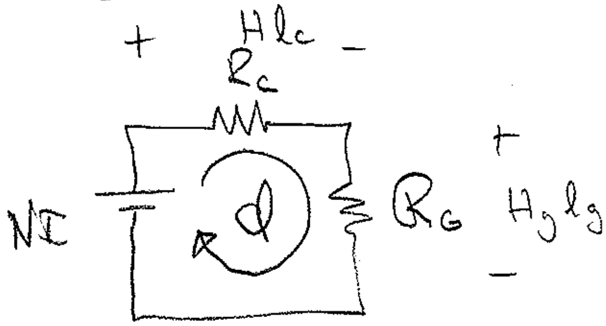
Faraday's Law $\Rightarrow$ voltages created in the iron core $\Rightarrow$ creates eddy currents.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \phi}{\partial t} + \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

Transformers use laminated cores.

No eddy currents in D.C.

Lab Experiment



$$\mathcal{R}_c = \frac{l_c}{\mu_c A_c}$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g}$$

Not a constant

Not a good way to deal with this since core is non linear.

So $\oint \vec{H} \cdot d\vec{l} = NI$.

$H_g l_g + H_c l_c = NI$ $\qquad$ $\underbrace{\phi \mathcal{R}_g = H_g l_g}$

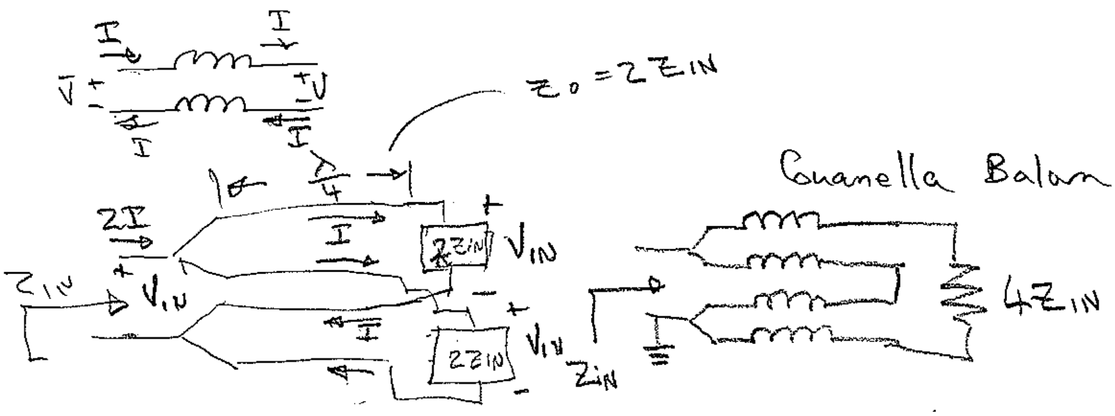
$B_g = \mu_0 H_g$

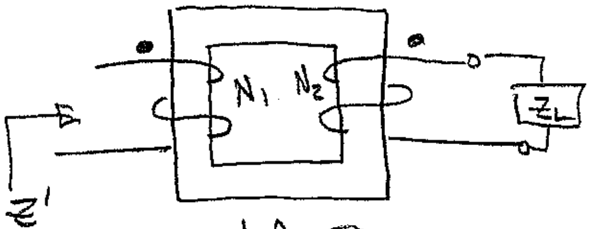
Measure B_g & make B-H curve.

Include fringing $A_g = (\sqrt{A_c} + g)^2$

Now Rod does a load line analysis to do a graphical solution. Get his notes.

Transformers





Ideal Transformers

No core losses

No leakage flux

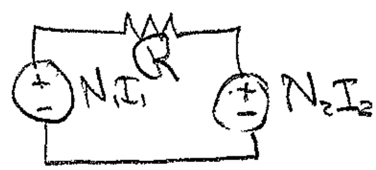
$\mathcal{R} \to 0 \Rightarrow$

No copper losses

Core does not saturate

$$\left.\begin{aligned} v_1 &= N_1 \frac{d\phi}{dt} \\ v_2 &= N_2 \frac{d\phi}{dt} \end{aligned}\right\} \quad \frac{V_1}{V_2} = \frac{N_1}{N_2} \equiv a$$

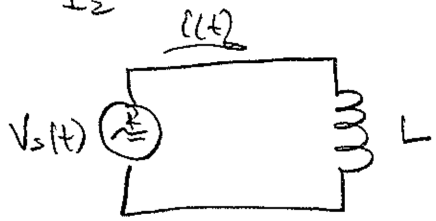
$$N_1 I_1 - N_2 I_2 = \cancel{\mathcal{R}}\phi = 0$$



$$\frac{I_1}{I_2} = \frac{1}{a}$$

$\vec{B} = \mu \vec{H}$

$$\frac{V_2}{I_2} = Z_L = \frac{1}{a^2} Z'$$



$$v = L\frac{di}{dt} \qquad \phi = Li$$

$$i = \frac{N\phi}{L} = \frac{\cancel{N}}{L} \frac{V_m}{2\pi f \cancel{N}} \sin(\omega t)$$

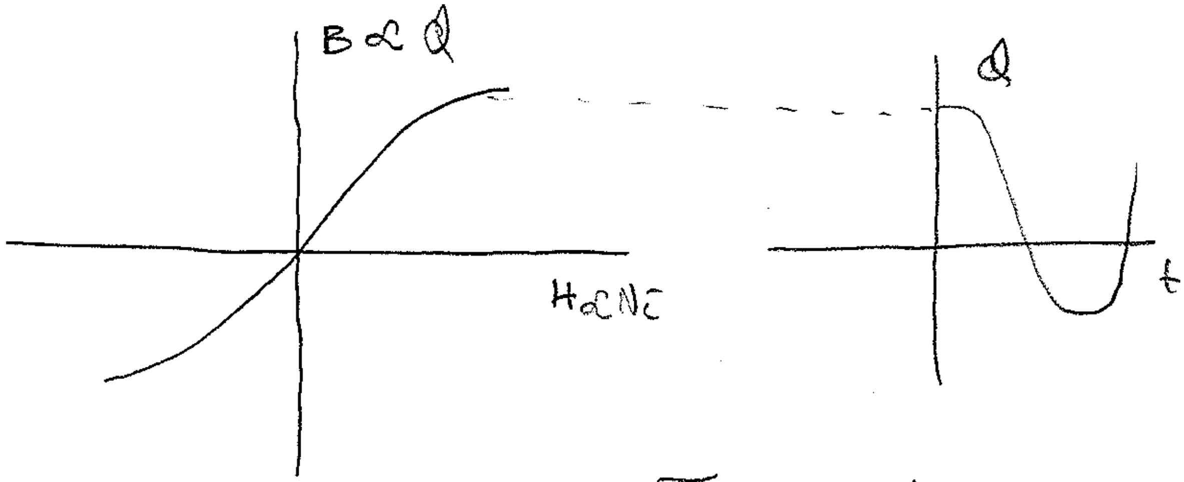
$$i = \frac{V_m}{\omega L} \sin(\omega t)$$

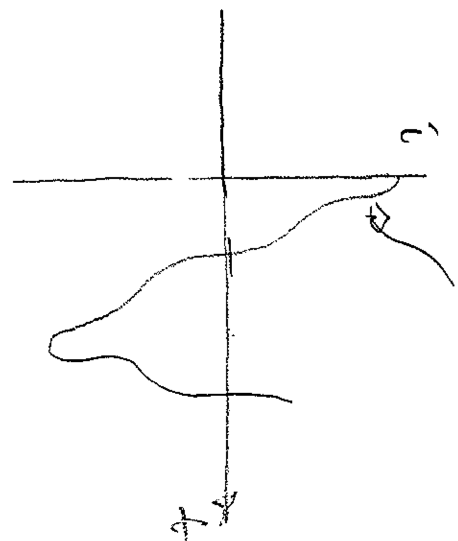
If $\phi = \phi_m \sin(\omega t)$

$$v = \underbrace{N\phi_m 2\pi f}\cos(\omega t)$$

$$V_m = N\phi_m(2\pi f)$$

$$V_{rms} = \frac{2\pi f N\phi_m}{\sqrt{2}} = 4.44 f N\phi$$

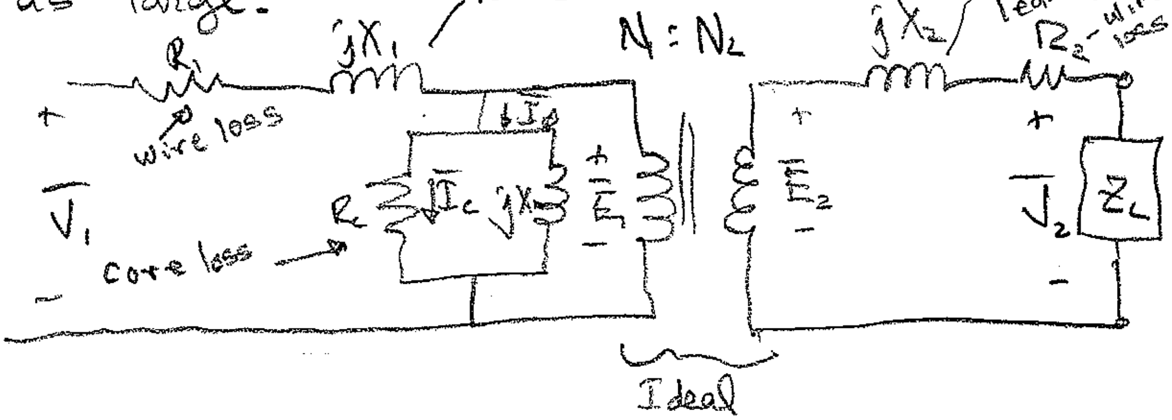




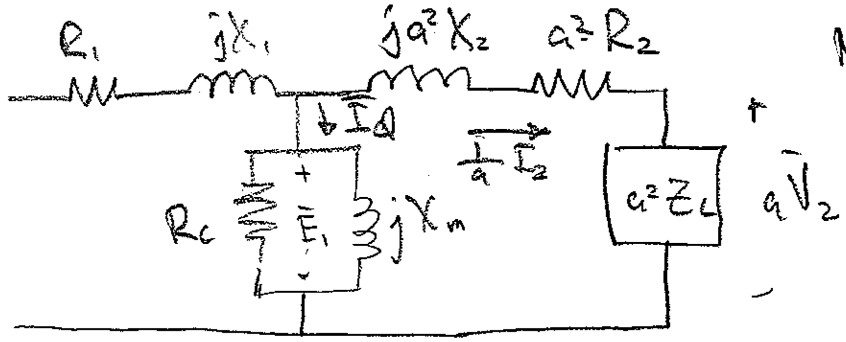
This implies a non sinusoidal current for a sinusoidal voltage.

Large third harmonic.

If we have a load the back flux reduces the peak flux so that the third harmonic distortion is not as large.

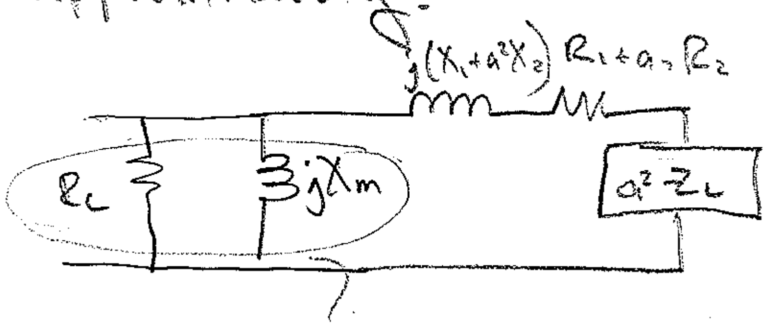


$$\frac{N_1}{N_2} \equiv a$$



Move to primary. You can move to secondary as well.

Because $|(R_1 + jX_1)| << |(R_c || jX_m)|$ this is approximately:



OK for Z_L fairly large ☺

Sometimes these are ignored as well.

Rod did 4.11 & 4.17.

$$\text{Voltage Regulation} = \frac{V_{2NL} - V_{2FL}}{V_{2FL}} \times 100\%$$

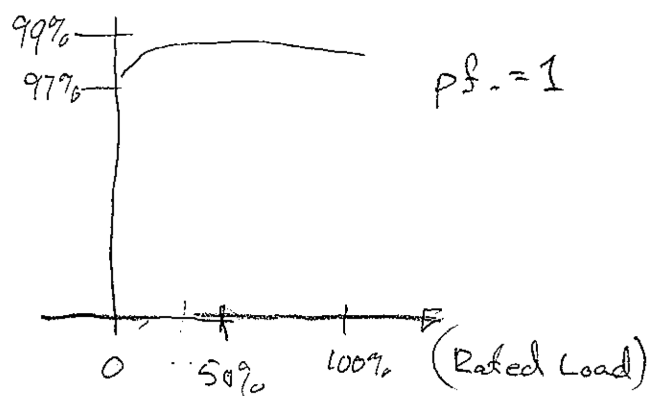
Rod did an example computing the model parameters from open & short circuit tests. He used the approximate model ☺ above. See page 232 & following in text.

Rod did 4.8. Book assumes sc test on the high voltage side. We didn't do that in lab.

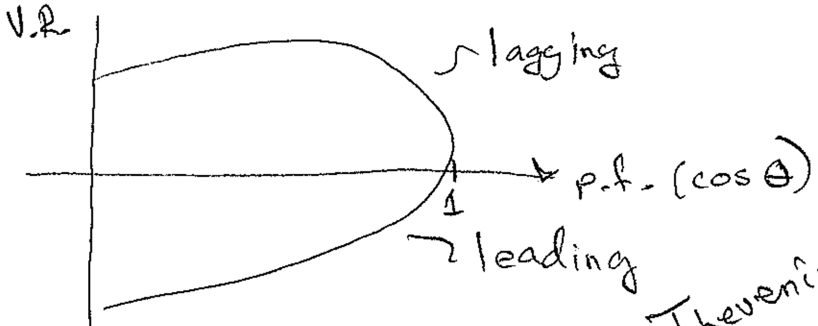
Rod does example just like lab analysis in class. He always uses ☺ now.

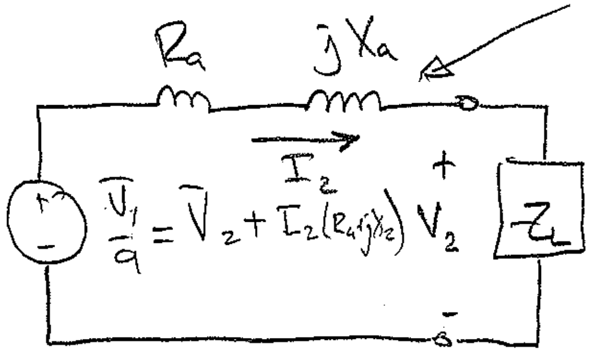
It is easier to assume the secondary is

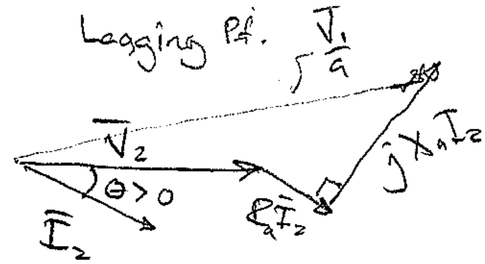
operating at rated load.
What happens to efficiency as the load decreases?

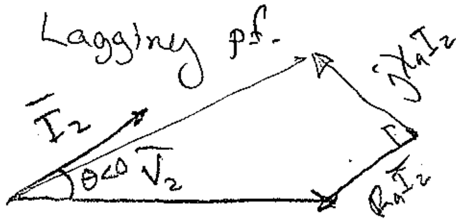


Rated Load VR
pf = 1 , 1.25%
pf = .8 lag , 2.1%
pf = .8 lead, −.083%







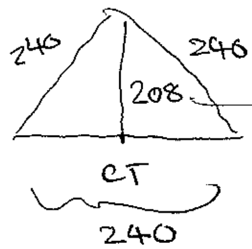


⟹ Negative voltage regulation

Induction Motors are really transformer.
Rod did several practice exam problems.
The exam is next day.

Transformers in Three Phase

Y-Y , Δ-Δ, Y-Δ, Δ-Y



Wild Leg

$V I C R \Theta \rightleftarrows$ Electrial System $\underset{\xleftarrow[\text{Generator}]{}}{\xrightarrow[\text{Field}]{\text{Motor}}}$ Mechanical Syst. ↑ loss

↘ loss

$W_e = \int v i \, dt$

$dW_m = F\,dx \qquad \frac{dW}{dt} = p = F\frac{dx}{dt} = Fv$

$dW_m = T\,d\theta \qquad \frac{dW}{dt} = T\frac{d\theta}{dt} = T\omega \qquad p = T\omega$ or vi

$v = N\frac{d\phi}{dt} = \underbrace{L\frac{di}{dt}}_{\text{if } L \text{ doesn't change}} \qquad v = \frac{d\lambda}{dt} \qquad (\lambda = Li)$

L changes as the magnetic circuit changes so $v = \frac{d\lambda}{dt}$ is better.

$W_f = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dv = \int_0^t \frac{d\lambda}{dt}\, i\, dt = \int_0^t \underbrace{i\,\frac{d(Li)}{dt}\,dt}_{i\,d\lambda}$

$= \frac{1}{2}Li^2$ if L doesn't change

Electrical energy = Mechanical Energy + Field Energy

$dW_e = dW_m + dW_f$

$dW_e = vi\,dt = i\frac{d\lambda}{dt}dt = i\,d\lambda$

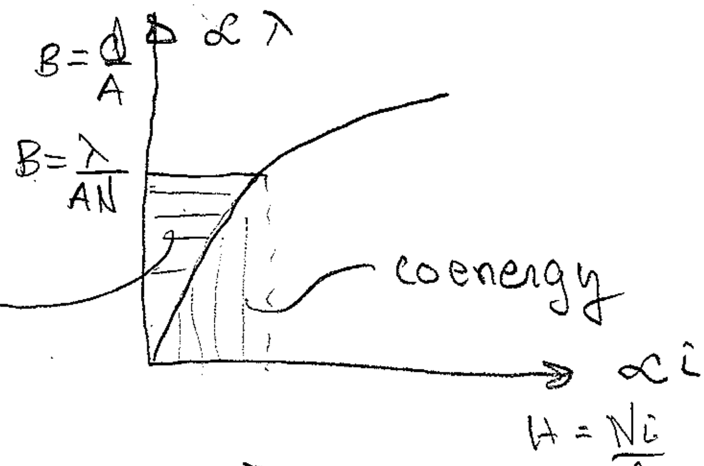
$dW_f = dW_e - dW_m$

$dW_f = i\,d\lambda - T\,d\theta$

Blocked Rotor

$dW_f = i\,d\lambda$

$N\phi = \lambda$



$$W_f = \int \vec{B}\cdot\vec{H}\,dv = \int i\,d\lambda$$

$$W_f = \lambda i - \underbrace{\int \lambda\,di}_{\text{Co energy}}$$

If $\vec{B} = \mu\vec{H}$ then the energy in the field is equal to the co-energy.

$$W_f = \int i\,d\lambda = \int i\,d(N\phi)$$

$$= \int Ni\,d\phi = \int \underbrace{\mathcal{F}}_{\text{MMF}}\,d\phi = \int \mathcal{F}\,d\left(\frac{\mathcal{F}}{\mathcal{R}}\right) = \frac{1}{2}\frac{\mathcal{F}^2}{\mathcal{R}}$$

$$= \int \mathcal{R}\phi\,d\phi = \frac{1}{2}\mathcal{R}\phi^2 \qquad \mathcal{F} = Ni = \mathcal{R}\phi$$

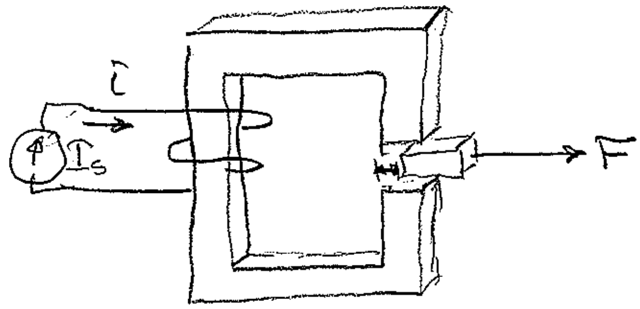
$$dW_f = dW_e - dW_m$$

$$\underbrace{\frac{dW_f}{dt}} = \underbrace{\frac{dW_e}{dt}}_{vi} - \underbrace{\frac{dW_m}{dt}}_{T\omega \text{ or } F_x v_x} \qquad W_f = f(\lambda,\theta) \text{ or } W_f(\lambda, x)$$

$$\frac{dW_f}{dt} = \underbrace{\frac{dW_f}{d\lambda}\frac{d\lambda}{dt}}_{i\,\cancel{\frac{d\lambda}{dt}}\,v} + \underbrace{\frac{dW_f}{d\theta}\frac{d\theta}{dt}}_{-T\omega}$$

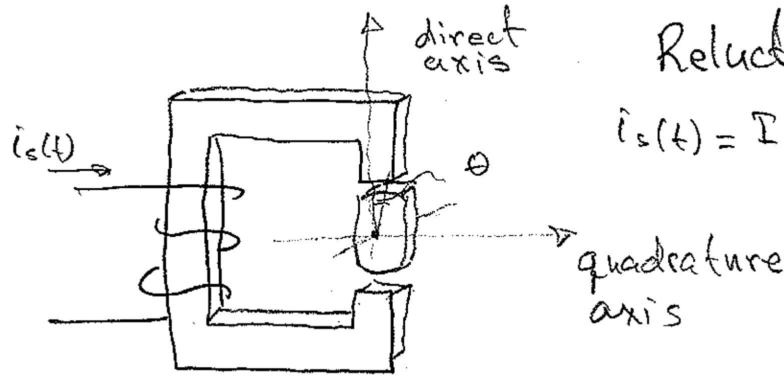
So $i = \frac{dW_f}{d\lambda}$ & $T = -\frac{dW_f}{d\theta}$.

or $F = -\frac{dW_f}{dx}$



Use "virtual work".

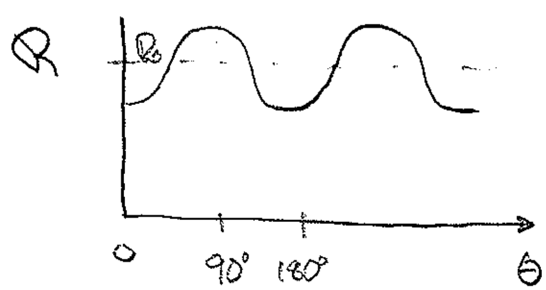
$$W = \int i \, d(iL) = i^2 L$$



Reluctance Motors

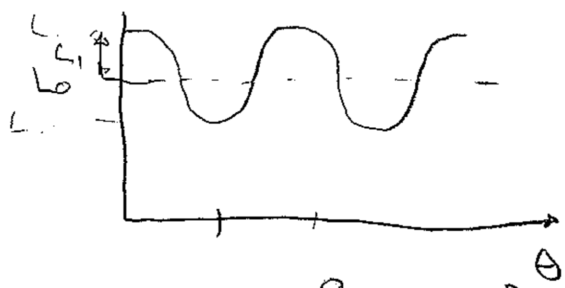
$$i_s(t) = I_s \sin(\omega t)$$

$$\mathcal{R} = \frac{l}{\mu A}$$



$$\mathcal{R}_\theta \cong \mathcal{R}_0 - \mathcal{R}_1 \cos(2\theta)$$

$$L = \frac{N^2}{\mathcal{R}}$$



$$L = L_0 + L_1 \cos(2\theta)$$

$$T = -\frac{dW_f}{d\theta}$$

$$W_f = \int i \, d(Li) \Longrightarrow \frac{dW_f}{d\theta} = \frac{i \, d(Li)}{d\theta}$$

$$= i^2 \frac{dL}{d\theta} + iL\frac{di}{d\theta}$$

$$\theta = \omega_m t + \delta$$

$$T = -\frac{dW_f}{d\theta} = -i^2 \frac{dL}{d\theta} - \cancelto{0}{iL\frac{di}{d\theta}}$$

Rod didn't do this. I did. Rod used $\frac{1}{2}Li^2$ & got $\langle T \rangle = -\frac{I_s^2 L_1}{4} \sin(-2\delta)$

$$T = +I_s^2 \sin^2(\omega t)\, L_1 2\sin(2\theta)$$
$$= +2LI_s^2 \sin^2(\omega t) \sin(2(\omega_m t + \delta))$$
$$= +2LI_s^2 \left(\frac{1-\cos(2\omega t)}{2}\right) \sin(2(\omega_m t + \delta))$$
$$= LI_s^2 \cos(2\omega t + \delta) - \frac{LI_s^2}{2}\left(\sin(2\omega t - 2\omega_m t - 2\delta) + \sin(2\omega t + 2\omega_m t + 2\delta)\right)$$

$\langle T \rangle = -\frac{LI_s^2}{2}\sin(-2\delta)$ when $\omega_m = \omega$.

Otherwise $\langle T \rangle = 0$.

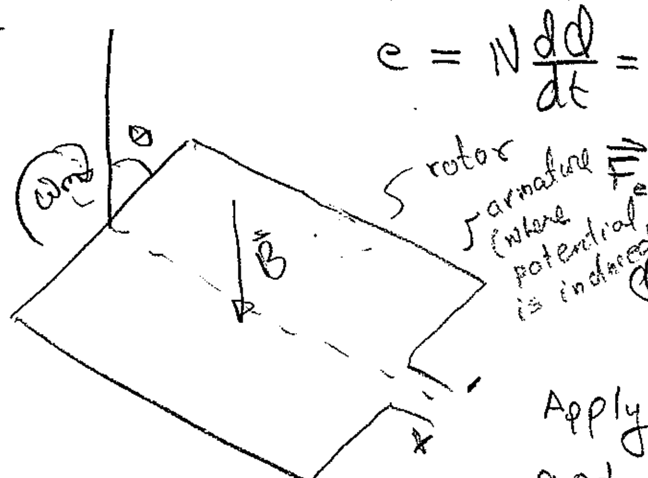
$\delta_{max} = 45°$

So the factor of two remains in the difference.

Why do we do all the current source excited examples?

DC Generator Basics

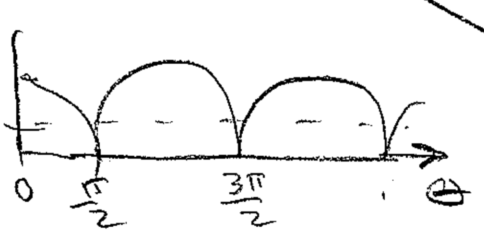
$$\phi = BA\sin\theta = BA\sin(\omega t)$$
$$e = N\frac{d\phi}{dt} = NBA\omega_m \cos(\omega_m t)$$



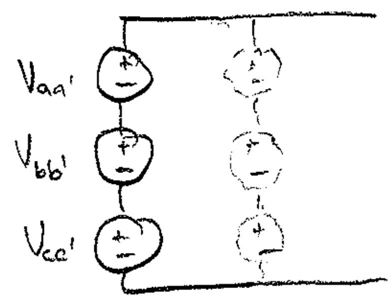
$\vec{F}_e = q\vec{v} \times \vec{B}$

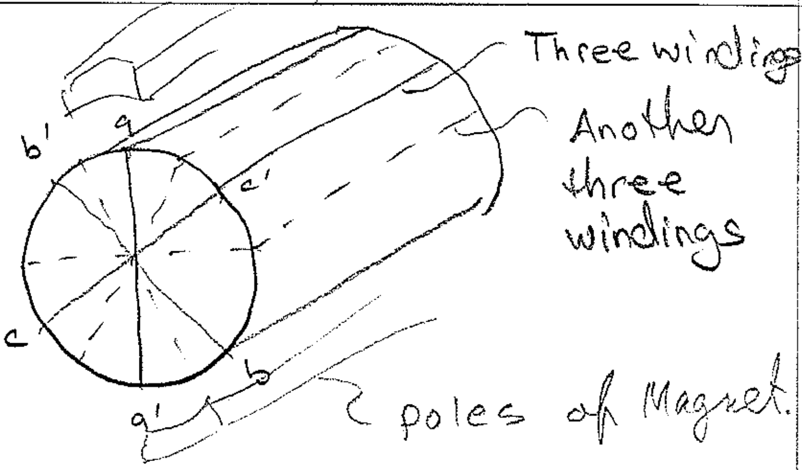
$\phi_d \equiv BA$

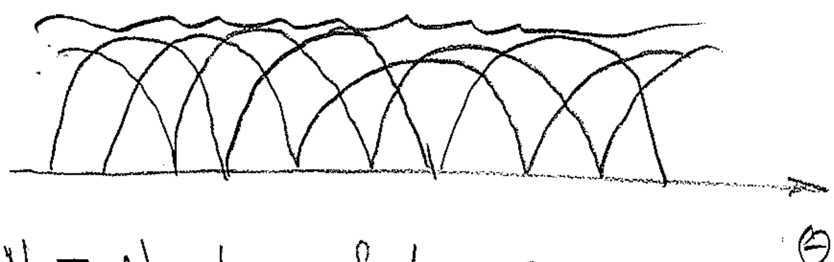
Apply slip rings to get a rectified output



$$\langle e \rangle = \frac{2N\phi_d\omega_m}{\pi}$$







$N \equiv$ Number of turns

$Z \equiv$ " " conductors

$a \equiv$ " " parallel paths

$\langle e \rangle = E_a = \dfrac{Z \Phi_d \omega_m}{a \pi} \dfrac{p}{2} (3) \equiv$ Armature potential

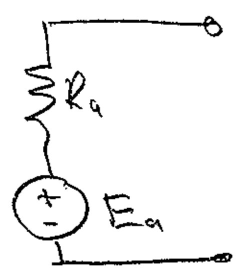
Put another pole pair to double the output.

$p \equiv$ # magnetic poles

$\underbrace{E_a \equiv K_a \Phi_d \omega_m}_{\text{Motor speed Equation}}$ where $K_a = \dfrac{Zp}{2a\pi} \equiv$ armature constant

As a motor:

$\omega_m = \dfrac{E_a}{K_a \Phi_d}$ Run away motor.

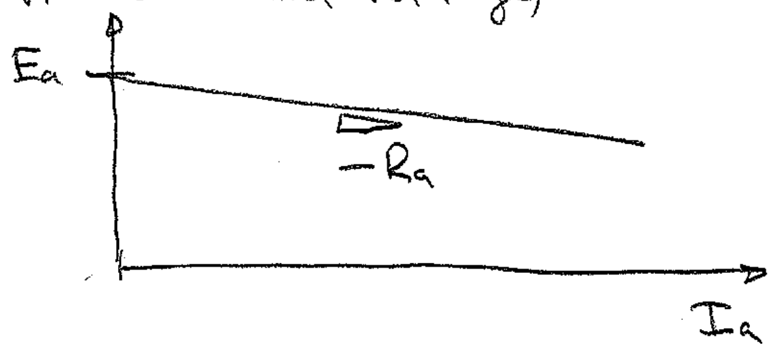


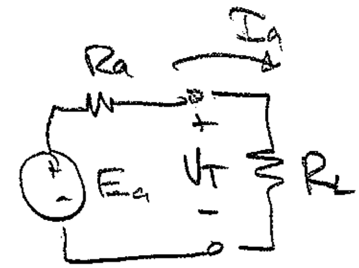
Armature Model

Field Model

Separately Excited Generator

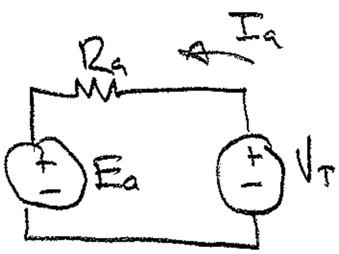
V_T (terminal voltage)



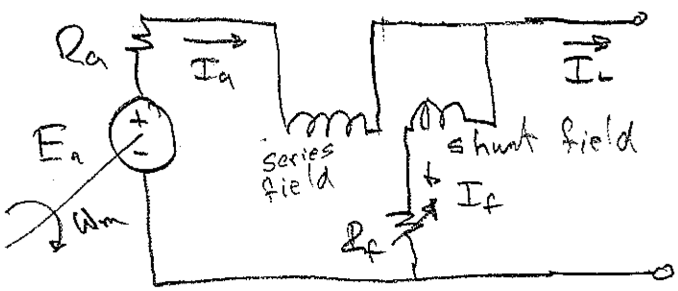


Motor

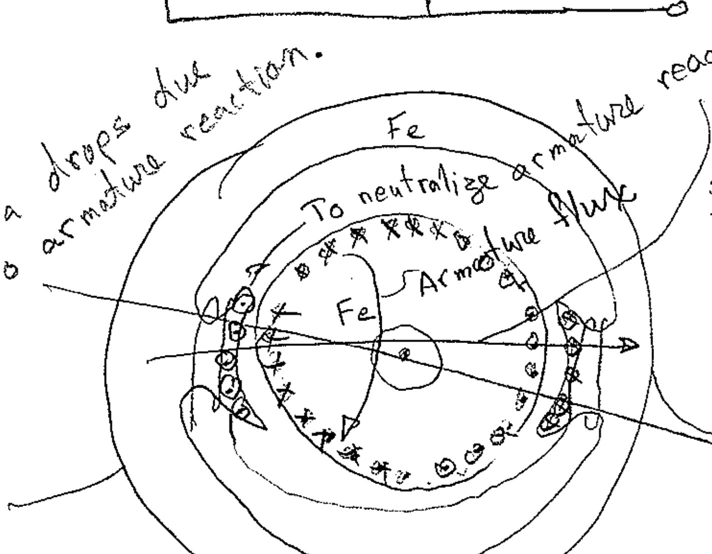
Current changes direction.

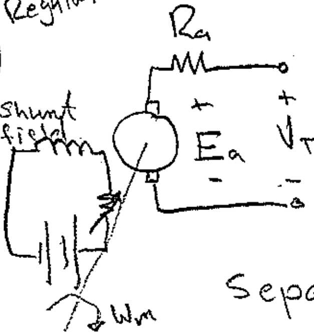


R_a includes series field, etc.



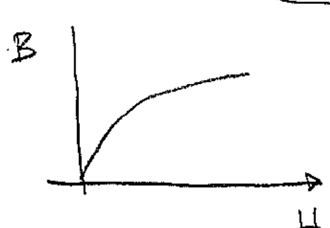
self excited Generator

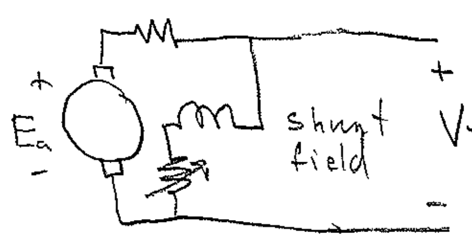




Separately Excited

Self Excited

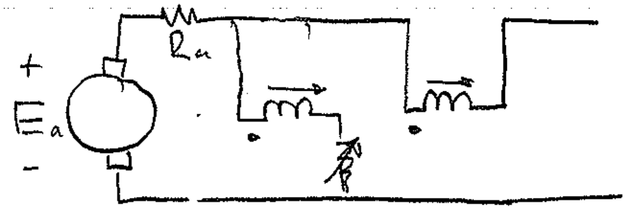




$\Rightarrow$ Reduced direct axis flux due to Armature reaction. The more armature current you draw the lower the flux

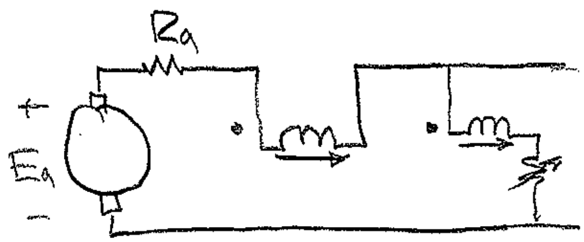
In motors the armature reaction causes the speed to increase under load.

Compound Self Excited Motor



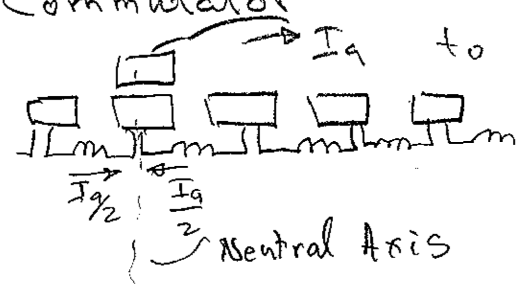
Short Shunt Cumulatively Compound Generator

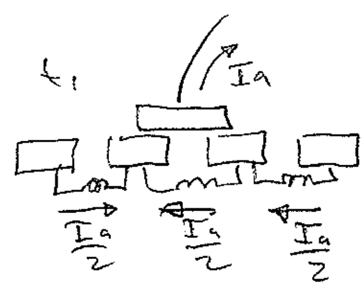
You can do a Differential Compound Generator but it has poor voltage regulation.

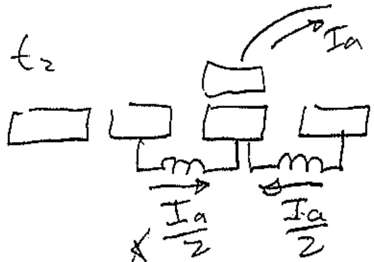


Long Shunt Cumulatively Compound Generator

Commutator





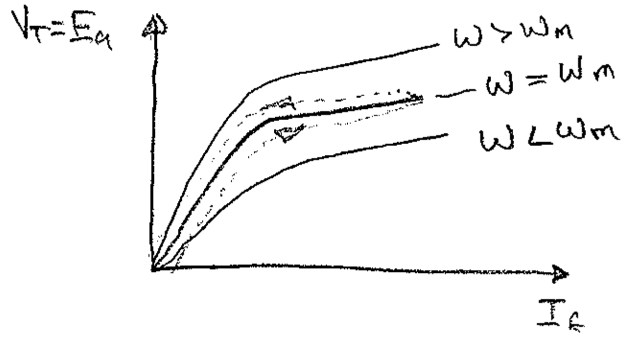


Somehow adding the commutating poles induces a voltage that counteracts the voltage from switching.

This suddenly changed directions

$v = L\frac{di}{dt} \Rightarrow$ arcing + pitted

No Load $\omega_m = \text{constant}$



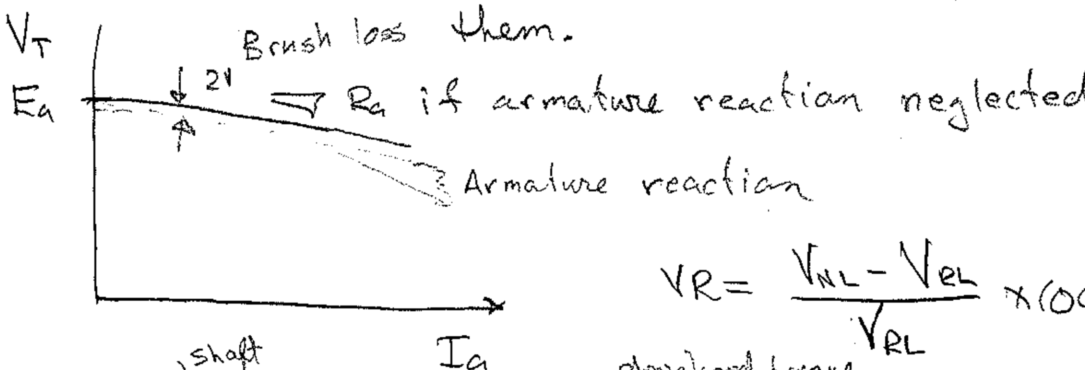
Magnetization Curve

Easy to measure.

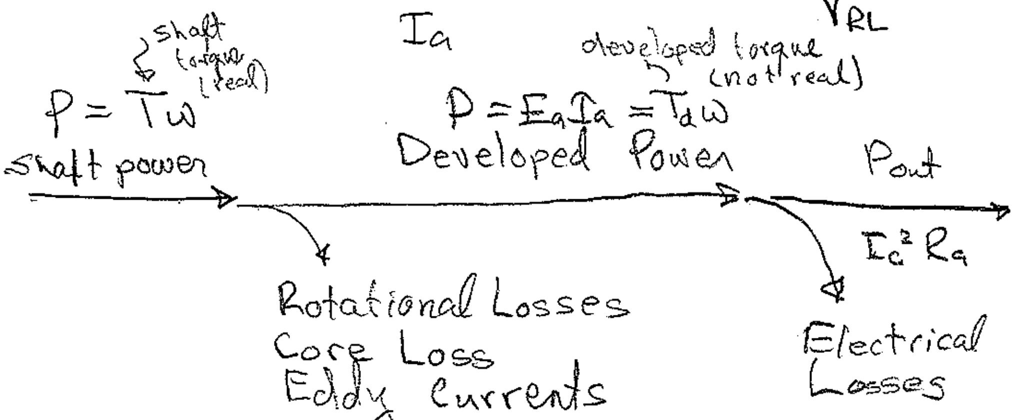
$$\frac{E_{a_1}}{E_{a_2}} = \frac{K_a \Phi_d \omega_{m_1}}{K_a \Phi_d \omega_{m_2}}$$

$$\frac{E_{a_1}}{E_{a_2}} = \frac{\omega_{m_1}}{\omega_{m_2}}$$

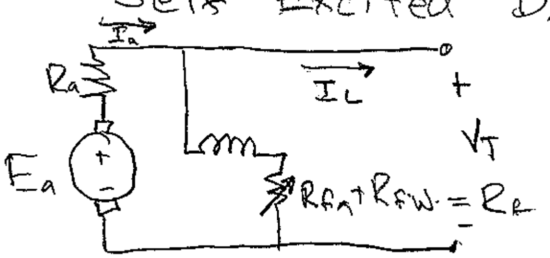
Commutator loses about 2V across them.



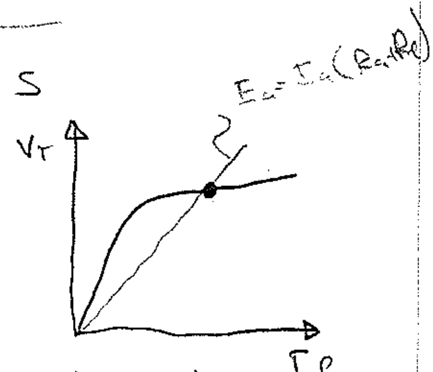
$$VR = \frac{V_{NL} - V_{RL}}{V_{RL}} \times 100\%$$



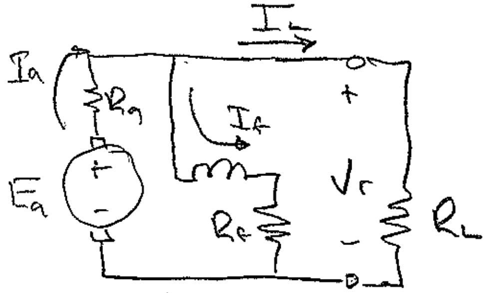
Self Excited D.C. Generators



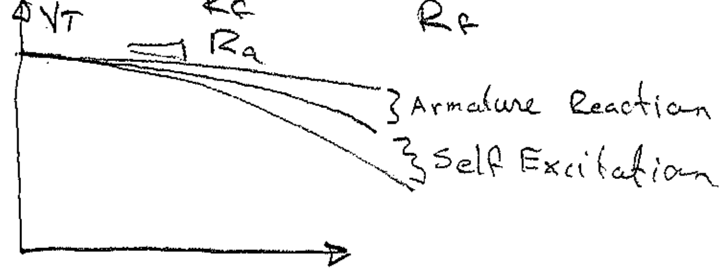
$$I_f = \frac{E_a}{R_a + R_f}$$
No Load



Based on residual flux. Only works when residual flux is aiding.



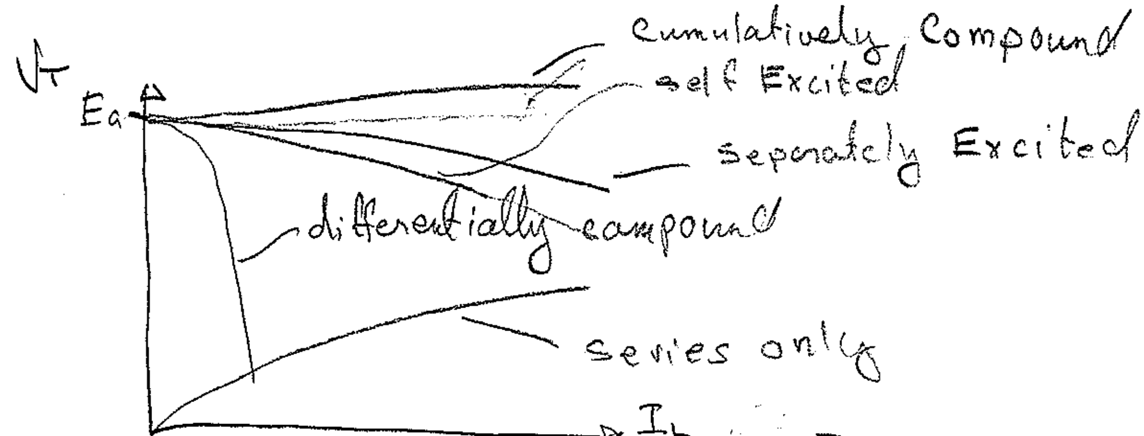
$$I_f = \frac{V_T}{R_f} = \frac{E_a - R_a \overbrace{I_a}^{I_f + I_L}}{R_f}$$

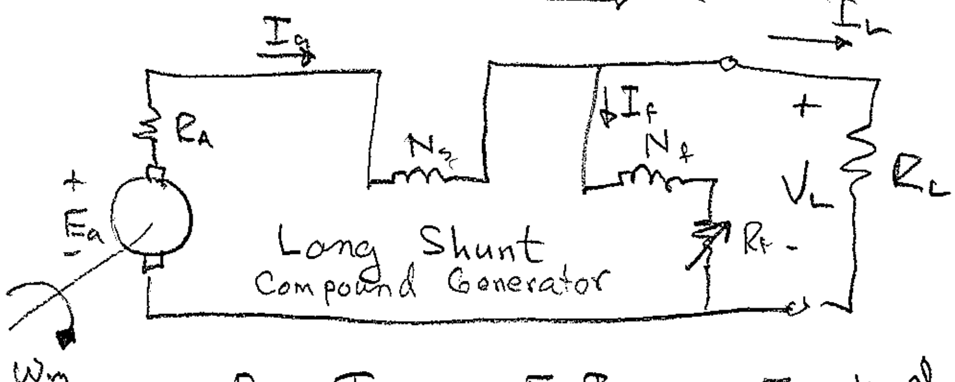


$E_a = K_a \phi_d \omega_m$

Rod did Example 5.6

To measure Protational loss drive the machine as a motor & $P_{in} = P_{Loss}$ assuming there is no mechanical load.





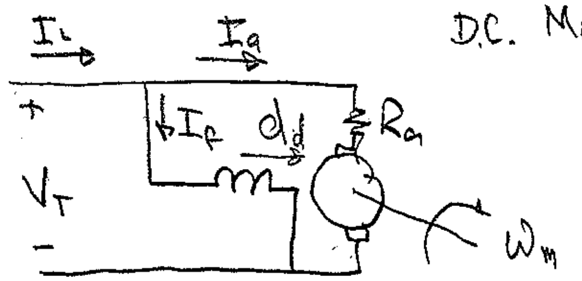
$P_d = T_d \omega_m = E_a I_a \qquad E_a = K_a \phi_d \omega_m$

$\Rightarrow T_d = K_a \phi_d I_a$

Rod did an example designing a series winding to produce flat regulation

Rod did #42 as an example.

Shunt Field D.C. Motor



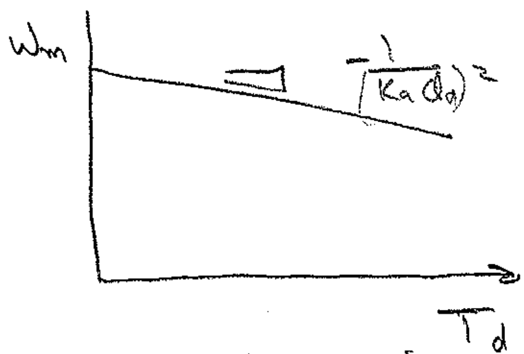
$E_a = K_a \phi_d \omega_m$

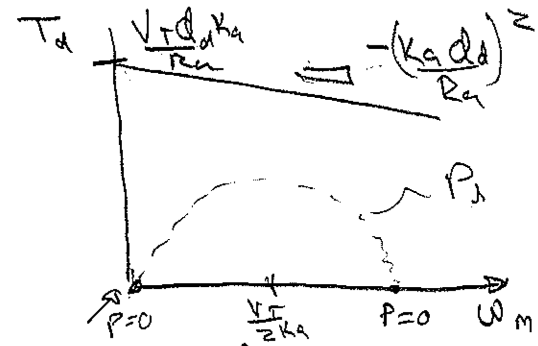
$E_a = V_T - I_a R_a = K_a \phi_d \omega_m$

$T_d = K_a \phi_d I_a \Rightarrow I_a = \dfrac{T_d}{K\phi_d}$

$I_a E_a = T_d \omega_m$

$$\omega_m = \frac{V_T}{K_a\phi_d} - \frac{I_a R_a}{K_a \phi_d} = \frac{V_T}{K_a\phi_d} - \frac{T_d R_a}{(K_a\phi_d)^2}$$





$$T_d = -\left(\omega_m - \frac{V_T}{K_a\phi_d}\right)\frac{K_a^2\phi_d^2}{R_a} = -\frac{K_a^2\phi_d^2}{R_a}\omega_m + \frac{V_T K_a \phi_d}{R_a}$$

$$P_d = T_d\omega_m = \frac{V_T K_a \phi_d}{R_a}\omega_m - \frac{K_a^2\phi_d^2\omega_m^2}{R_a}$$

$P_d = 0$ when $\omega_m = 0$ + when $\omega_m = \dfrac{V_T}{K_a\phi_d}$

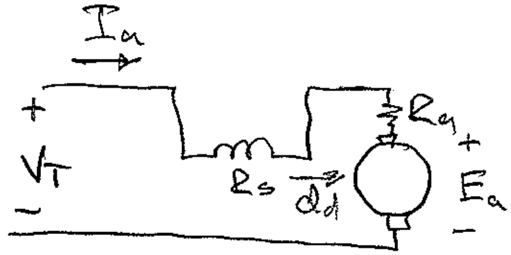
Find where in ω_m, P_d is max.

$$\frac{dP_d}{d\omega_m} = 0 \Rightarrow \omega_m = \frac{V_T}{2K_a\phi_d}$$

$$P_{d\,max} = \frac{V_T^2}{2R_a} - \frac{K_a^2\phi_d^2}{R_a}\frac{V_T^2}{4K_a^2\phi_d^2} = \frac{V_T^2}{4R_a}$$

Watch out for starting current.
" " " losing flux.

Series Field DC Motor



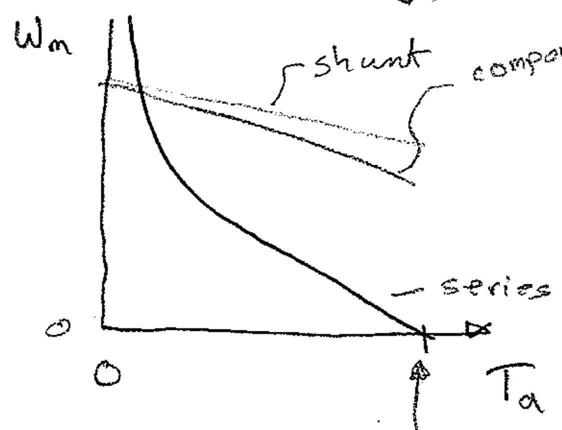
$$E_a = V_T - I_a(R_a+R_s)$$
$$= K_a \phi_d \omega_m$$
$$\mathcal{F} = I_a N_s = \mathcal{R}\phi_d$$

$$\omega_m = \frac{V_T - I_a(R_a+R_s)}{K_a \phi_d} = \frac{V_T}{K_a\phi_d} - \frac{\cancel{I_a}(R_a+R_s)}{\cancel{\phi_d}} = \frac{V_T}{K_a\phi_d} - \underbrace{\frac{\mathcal{R}}{N_s}(R_a+R_s)}_{\text{constant}}$$

$$\frac{\mathcal{R}}{N_s} = \frac{I_a}{\phi_d}$$

$$T_d = K_a\phi_d I_a = \frac{K_a\mathcal{R}}{N_s}\phi_d^2$$

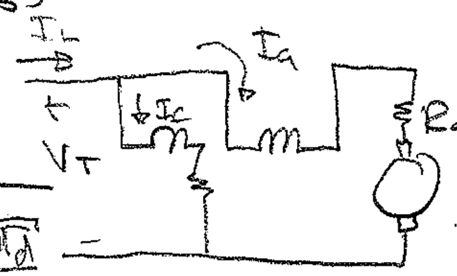
$$\text{so } \phi_d = \sqrt{\frac{N_s T_d}{K_a \mathcal{R}}} \Rightarrow \omega_m = \frac{V_T}{K_a\sqrt{T_d}}\sqrt{\frac{K_a\mathcal{R}}{N_s}} - \frac{\mathcal{R}(R_a+R_s)}{K_a N_s}$$



Unloaded, the series motor runs away.
Great starting torque.

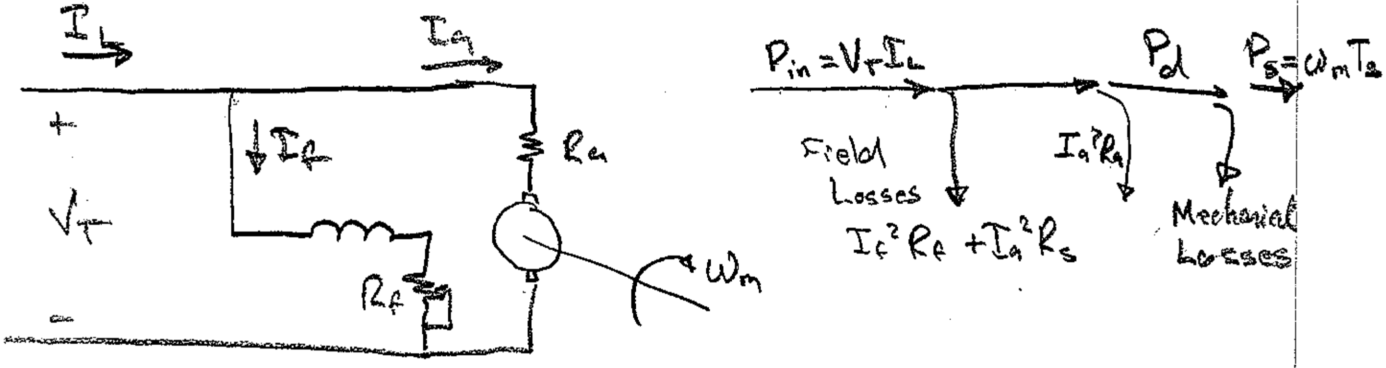
$$T_d = \frac{K_a V_T^2 N_s}{\mathcal{R}(R_a+R_s)}$$

Compound Motor



$$\omega_m = \frac{V_T}{K_a\left[\frac{N_f I_f}{2\mathcal{R}} + \sqrt{\left(\frac{N_f I_f}{2\mathcal{R}}\right)^2 + \frac{N_s T_d}{\mathcal{R}K_a}}\right]}$$

$$- \frac{T_d(R_a+R_s)}{K_a\left[2\left(\frac{N_s I_f}{2\mathcal{R}}\right)^2 + \frac{N_s T_d}{\mathcal{R}K_a} + \frac{N_f I_f}{\mathcal{R}}\sqrt{\left(\frac{N_f I_f}{2\mathcal{R}}\right)^2 + \frac{N_s T_d}{\mathcal{R}K_a}}\right]}$$



$E_a = K_a \phi_d \omega_m$

$\eta = \frac{P_s}{P_{in}}$

$P_d = E_a I_a$

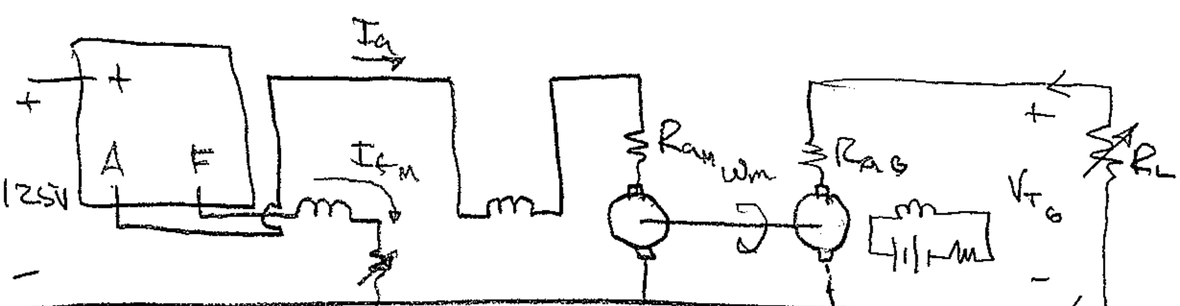
$T_d = \frac{P_d}{\omega_m} = \frac{K_a \phi_d \omega_m I_a}{\omega_m}$

Rod did an example finding η, ω_{FL}.
Rod did another example with magnetization curve & armature reaction (and without AR)

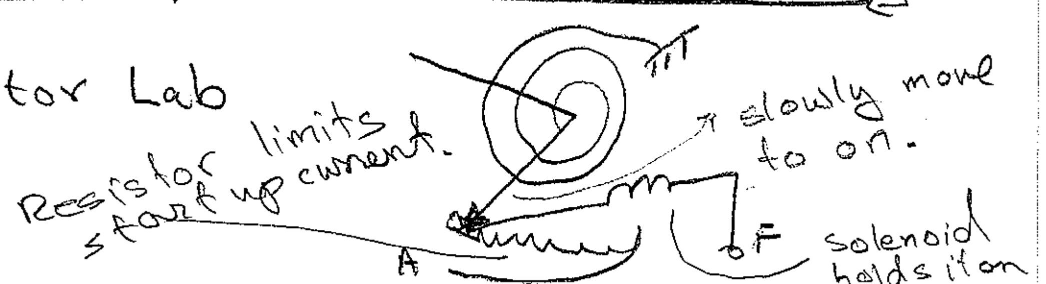
$I_{fe} = I_f - \frac{k_{ar} I_a}{N_f}$

$\mathcal{F} = N_f I_f - \underbrace{K_a I_a}_{\text{Armature reaction}}$

Rod did another example with a cumulatively compound motor short shunt.
Rod did 6.23 from the book.

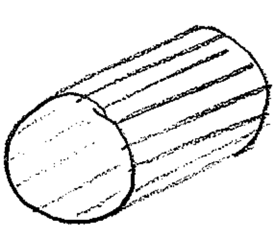


DC Motor Lab

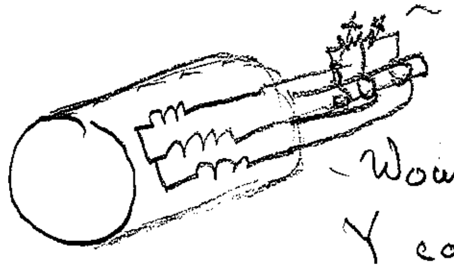


Rod did a series motor example.

Induction Motors



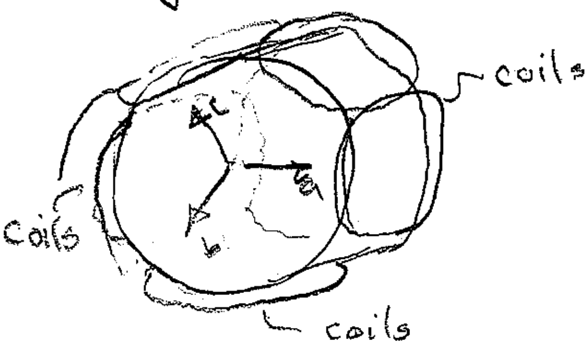
Squirrel Cage Rotor



~ Primative form of speed control

Wound Rotor
Y connection usually

Armature is fixed with a rotating magnetic field.

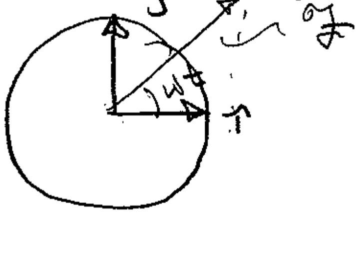


$$\mathcal{F}_a = NI\cos(\omega t)$$
$$\mathcal{F}_b = NI\cos(\omega t - 120°)$$
$$\mathcal{F}_c = NI\cos(\omega t - 240°)$$
$$\vec{\mathcal{F}} = \mathcal{F}_a\hat{i} + \mathcal{F}_b\left(-\tfrac{1}{2}\hat{i} - \tfrac{\sqrt{3}}{2}\hat{j}\right) + \mathcal{F}_c\left(-\tfrac{1}{2}\hat{i} + \tfrac{\sqrt{3}}{2}\hat{j}\right)$$

$$\vec{\mathcal{F}} = NI\left\{\left[\cos(\omega t) - \tfrac{1}{2}\cos(\omega t - 120°) - \tfrac{1}{2}\cos(\omega t - 240°)\right]\hat{i}\right.$$
$$\left. + \left[\frac{-\sqrt{3}}{2}\cos(\omega t - 120°) + \frac{\sqrt{3}}{2}\cos(\omega t - 240°)\right]\hat{j}\right.$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\vec{\mathcal{F}} = \frac{3}{2}\left[\cos\omega t\,\hat{i} + \sin(\omega t)\,\hat{j}\right] NI$$



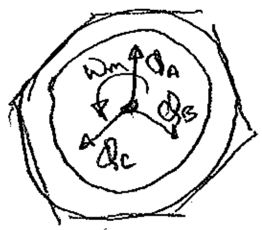
$n_s \equiv$ stator field speed

$$\underbrace{n_s = 60 f = \frac{60\,\omega}{2\pi}}\ \text{RPM}$$

For two poles/phase

In general $n_s = \frac{120 f}{P}$ where p is the number of poles/phase.

n	n_s
2	3600
4	1800
6	1200
8	900



Subscript 1 ⇒ Stator
" 2 ⇒ Rotor

$n_m \equiv$ mechanical speed

Assume $n_m = 0$ (blocked rotor or open rotor windings)

$\Phi_{A_1} = \Phi_m \sin(\omega_1 t)$

$\Phi_2 = K_2 \Phi_m \sin(\omega_1 t)$ K_2 is fraction of flux going through the rotor

$e_2(t) = N \frac{d\Phi_2}{dt} = \underbrace{2\pi f_1 N_2 K_2 \Phi_m}\cos(\omega_1 t)$

rms $E_2 = \frac{2\pi f_1 N_2 K_2 \Phi_m}{\sqrt{2}} = 4.44 f_1 N_2 \Phi_m$

rms $E_1 = 4.44 f_1 N_1 K_2 \Phi_m$ (back emf)

$$\frac{E_1}{E_2} = \frac{K_1 N_1}{K_2 N_2} \cong \frac{N_1}{N_2} = a$$

Like a transformer with an airgap.

$n_s = \frac{120 f_1}{P}$ (RPM) $\quad f_1 = \frac{P}{120} n_s$

$s \equiv \text{slip} \equiv \dfrac{n_s - n_m}{n_s} \leq 1 \qquad 1 \geq s > 0$

$n_m = (1-s)\, n_s$

$f_2 = \dfrac{P}{120}(n_s - n_m) = \dfrac{P}{120} s n_s$

$f_2 = \dfrac{P}{120} s \left(\dfrac{120}{P}\right) f_1 = s f_1$

The frequency on the secondary is different than the frequency on the primary.

$s = 1 \Rightarrow$ blocked rotor

$E_r = 4.44\, s f_1 N_2 k_2 \Phi_m \leftarrow$ (rms) rotating rotor

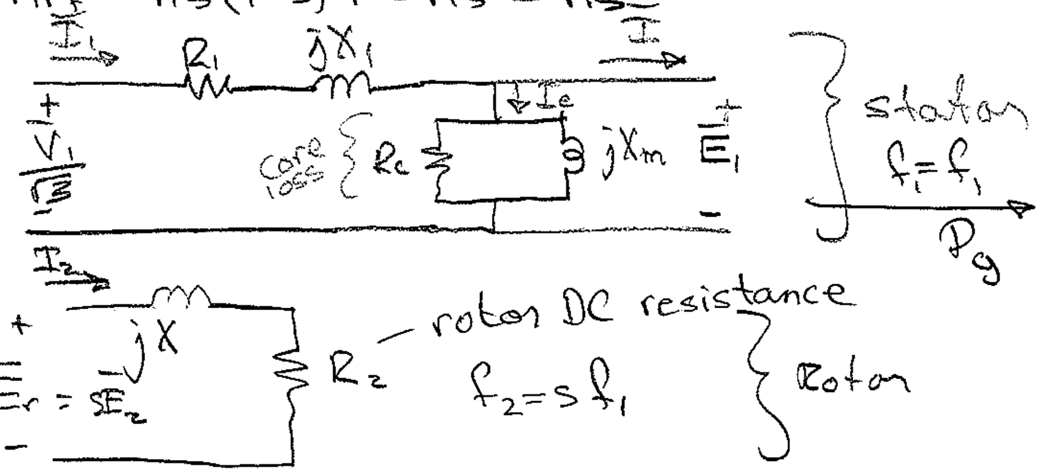
$= s E_2$

$n_{rf} \equiv$ speed of rotor field $= n_m + n_2$

$n_{rf} = n_s(1-s) + n_2$

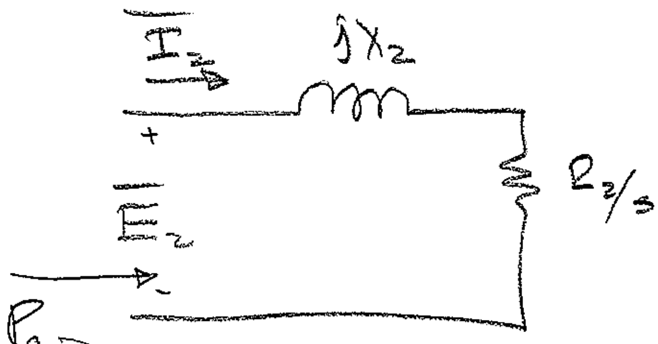
$n_2 = 60 f_2 = 60 s f_1 = s n_1 = s n_s$

$n_{rf} = n_s(1-s) + s n_s = n_s$



$jX = j2\pi s f_1 L = js X_2 \qquad \bar{I}_2 = \dfrac{s\bar{E}_2}{R_2 + jsX_2}$

$\bar{I}_2 = \dfrac{\bar{E}_2}{\frac{R_2}{s} + jX_2} \qquad P_{3cu} = 3 I_2^2 R_2$

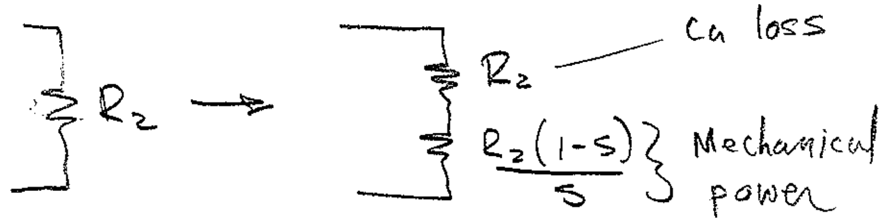


P_g ~ gap power

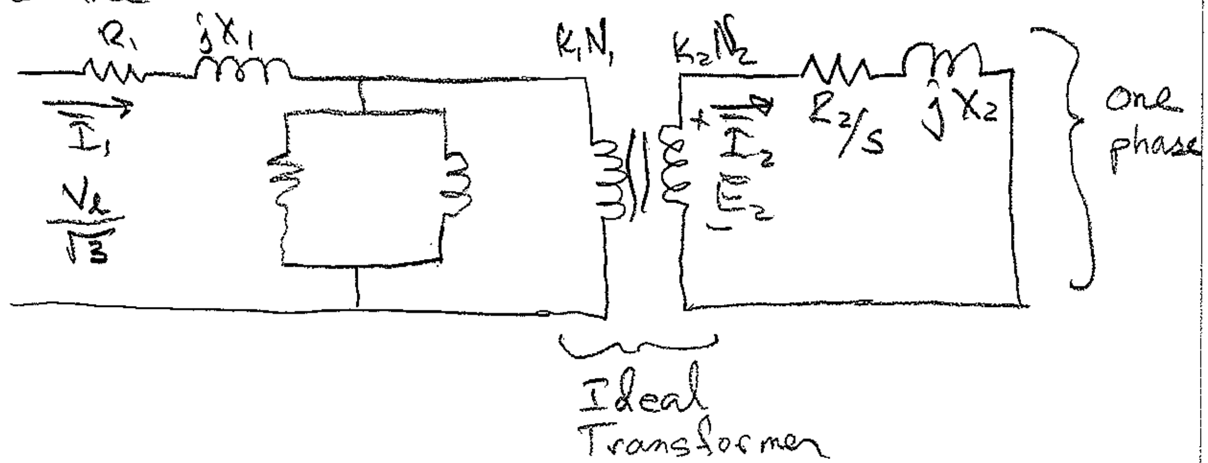
E_2 is at f_1 so this is combinable with the primary, by multiplying by a^2.

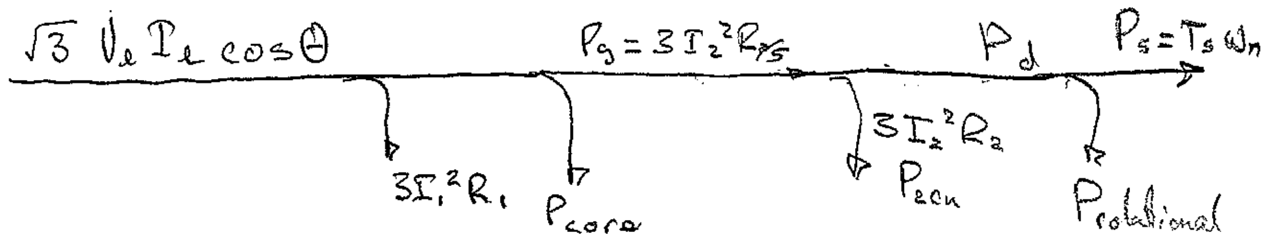
$$\underbrace{P = 3I_2^2 R_2/s}_{\text{cu loss \& mechanical power.}}$$

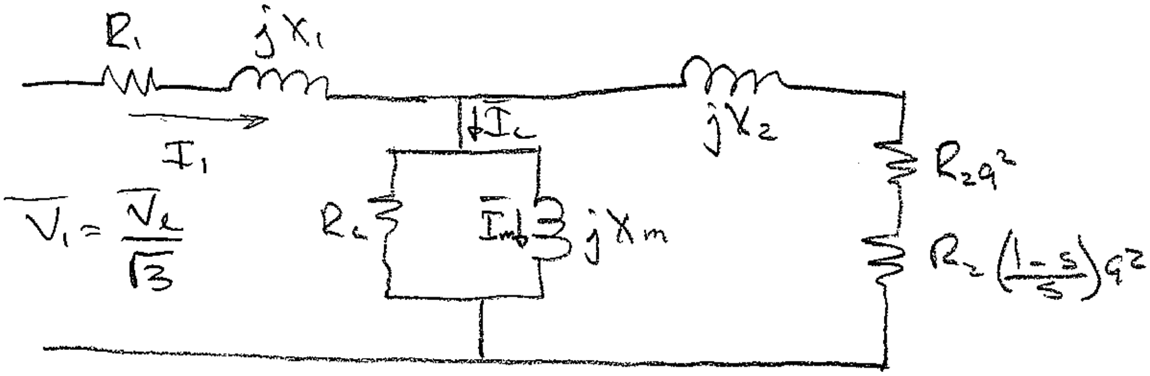
$$\frac{R_2}{s} = R_2 + \frac{R_2}{s}(1-s)$$



Put it all together now that the frequencies are the same.



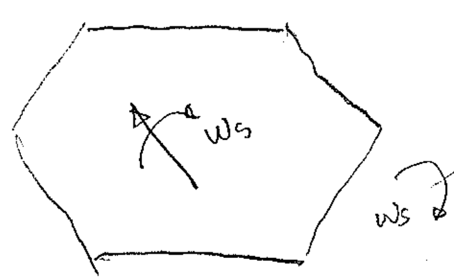


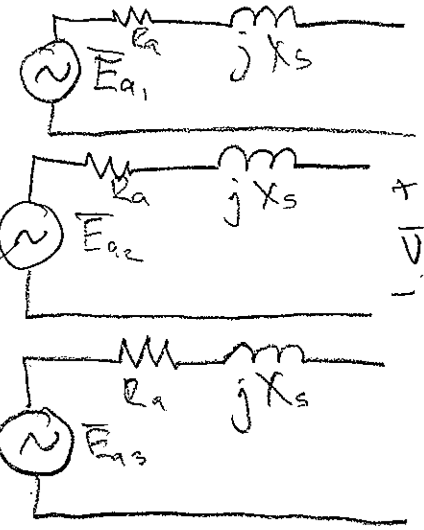


$R_c \gg X_m$ so often it is ignored.

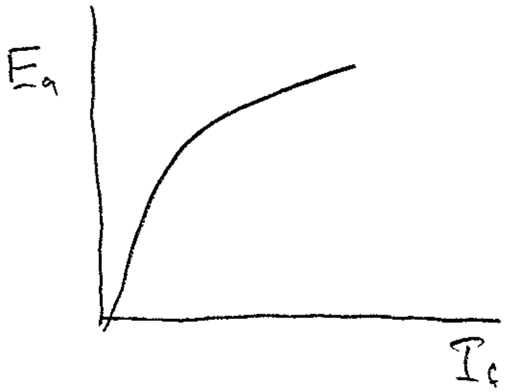
Rod did an example.
Rod did example 9.7 & 9.8.

Synchronous Machines (Chapter 8)
The armature is the same as an induction motor. The rotor is an electromagnet.
AC is generated via synchronous generators (alternators).
Look up three pase rectifiers
Magneto has a permanent magnet rotor.
Synchronous motor is started as an induction motor.





$$\bar{E}_{a_i} = \frac{j2\pi f \Phi N}{\sqrt{2}} K_i$$
$$= K\bar{\Phi}\omega$$



jX_s is from leakage reactance & armature reaction

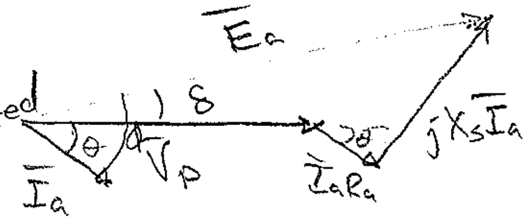
$E_a = K \phi \omega$

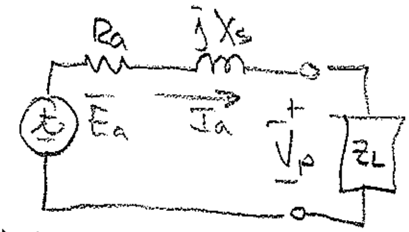
Generator action $\quad \bar{E}_a = \bar{V}_p + \bar{I}_a(R_a + jX_s)$

Motor Action $\bar{I}_a$ changes direction.

VR>0

over excited

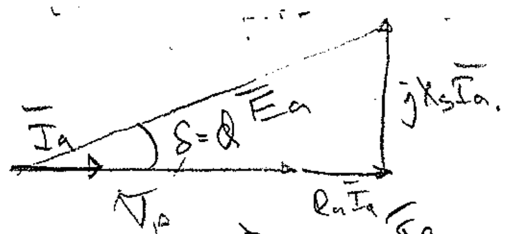




Leading power factor $\theta > 0$.

$Z_L = |Z_L| e^{j\theta}$

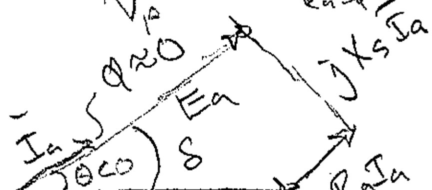
VR>0



$\phi = \delta$

$\theta = 0$

VR<0 or VR≥0



Lagging pf.
$\theta < 0$

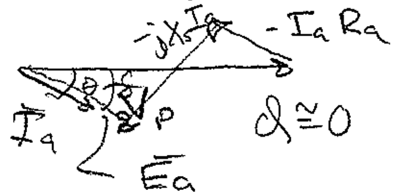
generator

under excited

VR ≡ voltage regulation.

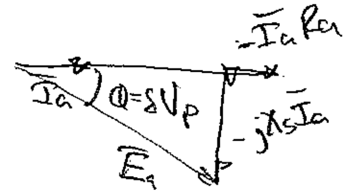
ϕ ≡ power factor as seen from $\bar{E}_a$.

$$\bar{E}_a = \bar{V}_p - \bar{I}_a(R_a + jX_s)$$ Motor ($\bar{I}_a$ changes direction)

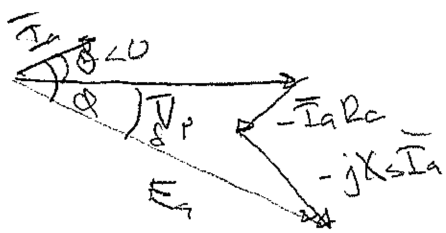


$\theta > 0$

under excited



$\theta = 0$



$\theta < 0$

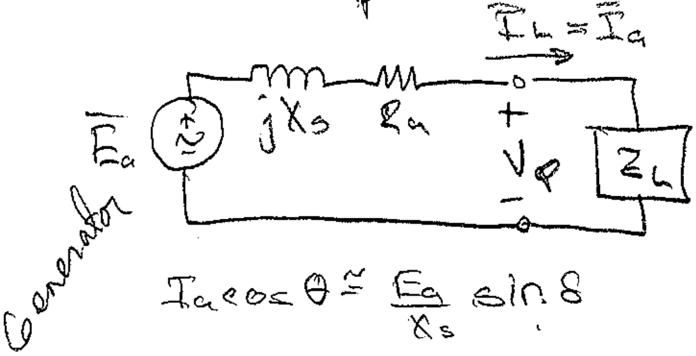
overexcited

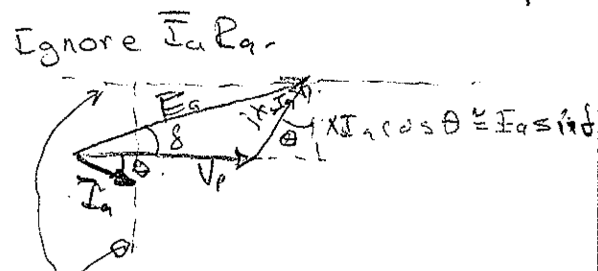
We can control the power factor by changing δ which changes the length of $\bar{E}_a$.

$P_o = 3V_\phi I_L \cos\theta$

$P_d = 3V_p I_L \cos\delta$

Ignore $\bar{I}_a R_a$.



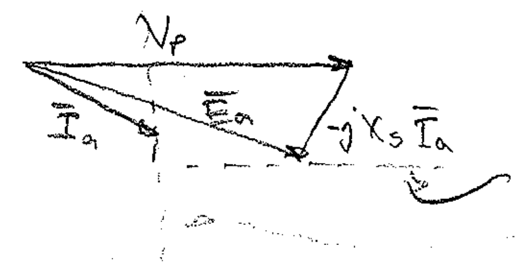


$I_a \cos\theta \cong \frac{E_a}{X_s} \sin\delta$

$E_a = K\phi\omega_m$

E_a doesn't change with P_L

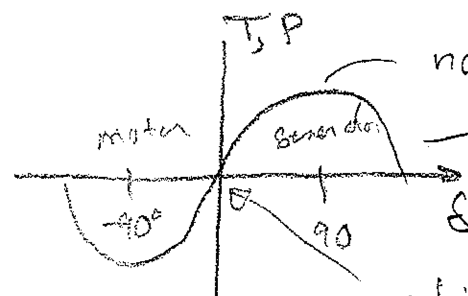
As a motor (ignoring $\bar{I}_a R_a$):



loci of constant power

change δ to move I_a + E_a along these loci.

$P_o = T\omega_s$



not stable

slipping poles

stiffer here

$$P_o = \frac{3V_p E_a \sin\delta}{X_s}$$

$\delta > 0 \Rightarrow$ Generator

$\delta < 0 \Rightarrow$ Motor

$$\text{stiffness} \equiv \frac{dP}{d\delta}$$

Lab machines are 4 pole so strobe shows $\delta/2$.

Rod did 8-1 as an example.

" " 8-5 " " "

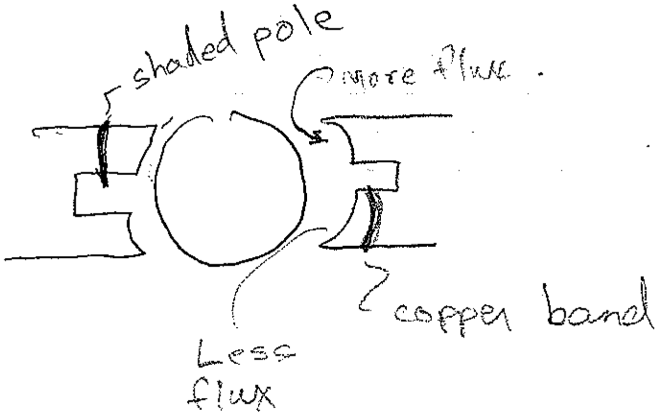
How do we start single phase motors?

1) One way to do it is an auxillary winding. One winding has large R & small X and one with small r & large X.

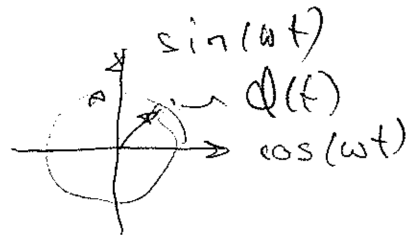
2) Use an auxillary winding with a parallel or series capacitor.

You need time & space quadrature.

Sometimes you remove the auxillary coil with a centrifugal switch.



Reluctance motors have reluctance that changes as a function of θ.



Measure field current on the synchronous machines by using the 10 A scale.

Strobes can induce epilyptic seasures.